

**ТЕОРИЯ ВЕРОЯТНОСТЕЙ,  
МАТЕМАТИЧЕСКАЯ СТАТИСТИКА  
И ИХ ПРИЛОЖЕНИЯ**

**PROBABILITY THEORY,  
MATHEMATICAL STATISTICS  
AND THEIR APPLICATIONS**

**Выпуск  
Number 2**



БЕЛОРУССКИЙ ГОСУДАРСТВЕННЫЙ УНИВЕРСИТЕТ

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СБОРНИК НАУЧНЫХ СТАТЕЙ

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**PROBABILITY THEORY,  
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THE COLLECTION OF SCIENTIFIC PAPERS

Number 2

МИНСК  
РИБШ  
2009

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# STATE BORDER CORRUPTION AND ILLEGAL MIGRATION: A MODEL

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В структуре распределения ресурсов, предложена модель нелегальной миграции с учетом пограничной коррупции, принимая во внимание западные страны СНГ. Предполагается, что гипотетическая власть пограничной службы старается минимизировать среднее число нелегальных пересечений госграницы. Это число линейно зависит от уровня коррупции в местных администрациях при оптимальном распределении бюджетных ресурсов и фиксированном числе пограничников.

*Keywords:* Illegal migration, irregular migration, people smuggling, illegal border crossings, syndicates, corruption, border administration, state border corruption

In recent years, a number of authors have pointed out the relationship between irregular migration and state border corruption. The critical variable seems to be the interaction between the governments and organised crime syndicates [1, p. 149]. There is a direct relationship between the regimes that organise and regulate migratory movements, and the scope of irregular migration [6, p. 78]. As syndicates have become more sophisticated and profitable – as a consequence of the higher demand and cost for their services – their capacity and means to corrupt have also grown [3, p. 121]. Corruption is now a key element in human smuggling and trafficking because it makes it easier to get the migrants across the borders [1, p. 149] and should therefore be viewed as an essential cog in the wheel of labour trafficking [5, p. 148]. Without employees at border checkpoints turning a blind eye, often after the payment of a significant sum of money, this form of organised crime could not proceed [7, p. 55]. Payment of bribes in money, goods, or kind can persuade an official to turn a blind eye to improper documentation or protection against scrupulous checking of vehicles, cargo holders, or vessels holding smuggled migrant workers [9, p.157].

In the Western Newly Independent States – WNIS - there is a lot of anecdotal evidence about state border corruption. As an example, it has been pointed out that Ukraine's ability to address irregular migration has been limited by widely acknowledged but as yet unsubstantiated corruption of consular, border guard, law enforcement and state officials [6, p. 79]. In addition, the results from a survey carried out in Ukraine in 2001 showed that many people were concerned about corruption within customs and border services [4, p. 12].

There is also actual evidence of corruption. The European Union's Border Assistant Mission to Ukraine and Moldova – EUBAM – points out that even though border corruption has been decreasing in recent years, it is still present in the two countries [2, p.13]. Measures have been taken to improve the situation. At the field level, EUBAM has been able to advice on im-

proving procedures in order to reduce the opportunities for corruption. A joint operation has been created in which border guards and the police have begun to operate more closely together. A special system, ASYCUDA, has been introduced in order to restrict direct payments made to customs officers. In addition, a directive has been implemented limiting the amount of money to be carried by officers and prohibiting the use of mobile phones [ibid].

In the Republic of Moldova, during the first 10 months of 2008, disciplinary measures were taken against 183 border officials, of which 36 were sacked for corrupt acts and behaviour. 10 files were passed on to the public prosecutor and 6 people were arrested. As a comparison, in the year before, 179 were punished and one person sacked. In 2006, 143 employees were fined and no-one sacked [9].

To date, the economic modelling of state border corruption is basically non-existent. One reason is that data more detailed than the above is extremely hard to come by, due to the clandestine nature of the subject and the authorities' lack of willingness to share information. Using data from court cases is a difficult approach as well, since there would be no way of adjusting the parameters in the model without knowing the shadow size of true cases and the prosecution intensity. However, for a purely theoretical context, there are several approaches that one could think of.

In a game theoretical setting, one approach would be to construct a regular auditing model with mixed strategy equilibria and regular observation. Contrary to Nash equilibria, such models do not predict that auditors react only to incentives faced by their counterpart, but they also react to own incentives (at least in a lab environment). See, for example [8]

Another possibility would be to study state border corruption within a framework of resource allocation. The following model assumes that the task of the border guard service is to minimise the number of illegal border crossings, in presence of corruption. We start out with the following designations:

$N$  - the general number of border guards,

$L$  - the number of oblasts protected by the Border Guard Service,

$N_i$  - the number of border guards in  $i$  - th oblast,  $\sum_{i=1}^L N_i = N$ ,

$\beta_i$  - the average number of migrants in  $i$  - th oblast, trying to cross the border with the help of corrupt border guards for the defined period,

$a_i$  - the level of corruption in  $i$  - th oblast,

$B_i$  - budgetary funds allocated to border protection in  $i$  - th oblast,

$B$  - general budgetary expenses allocated to the protection of the state border.

We assume that the level of corruption is inversely proportional to the level of budgetary investments. The higher the wage of the border guard, the less likely it is that he or she will be inclined to corrupt behavior. We therefore assume that

$$a_i = \frac{cN_i}{B_i}$$

where  $c$  is a coefficient of proportionality, as a first general approximation for all oblasts. Coefficient  $a_i$  is the share of corrupt border officials. Furthermore, it is clear that even in presence of corrupt border guards, a risk of disclosure still exists for the group of illegal migrants. It is therefore natural to introduce the probability  $p_i$  for a successful border crossing of the group of illegal migrants, which is proportional to the share of corrupt border guards. That is:



$$p_i = c_i a_i, \quad 0 < c_i < 1$$

One of the purposes of the Border Guard Service is to minimize the number of illegal migrants crossing the state border. Out of this purpose, a problem of optimally distributing budgetary funds to areas so as to minimize the general average of illegal migrants, arises. Mathematically, this can be formulated in the following way:

$$M = \sum_{i=1}^L \beta_i p_i = \sum_{i=1}^L \beta_i c_i \frac{N_i}{B_i} \rightarrow \min_{B_i}, \quad (1)$$

under constraints

$$\sum_{i=1}^L B_i = B \quad (2)$$

The magnitude  $M$  is the average number of illegal migrants who have successfully overcome the border. Having expressed  $B_i$  from the relationship (2) and having equated the partial derivatives  $M$  and  $B_i$  to zero,  $i=2, 3, \dots, L$ , we get

$$\frac{\partial M}{\partial B_i} = \frac{-\beta_i c_i N_i}{\left(B - \sum_{i=2}^L B_i\right)^2} - \frac{\beta_i c_i N_i}{B_i^2} = 0 \quad (3)$$

After reduction to a common denominator, we have

$$\frac{-\beta_i c_i N_i B_i^2 - \beta_i c_i N_i (B - \sum_{i=2}^L B_i)^2}{\left(B - \sum_{i=2}^L B_i\right)^2 B_i^2} = 0 \quad (4)$$

$i=2, 3, \dots, L$

The system (4) can be solved numerically. However, for the case  $L=2$  it can be solved analytically. In this case the equations of the system (4) are transformed into one quadratic equation:

$$k_1(N - N_2)B_2^2 - k_2N_2(B - B_2)^2 = 0,$$

where  $k_i = \beta_i c_i c$ ,  $i=1, 2$ .

Its positive solution looks as follows:

$$B_2 = \frac{-k_2N_2B + B\sqrt{k_1k_2N_1N_2}}{k_1N_1 - k_2N_2}.$$

That is, taking into account (2)

$$B_1 = \frac{k_1N_1B - B\sqrt{k_1k_2N_1N_2}}{k_1N_1 - k_2N_2}.$$

Thus the average number of illegal migrants who have successfully overcome the border equals

$$M_{\min} = k_1N_1 \frac{k_1N_1 - k_2N_2}{k_1N_1B - B\sqrt{k_1k_2N_1N_2}} + k_2N_2 \frac{k_1N_1 - k_2N_2}{-k_2N_2B + B\sqrt{k_1k_2N_1N_2}}$$

We investigate the dependence of this magnitude on the level of corruption. Considering that

$$a_i = \frac{cN_i}{B_i} \text{ and } k_i = \beta_i c_i c, \text{ we see that } M_{\min} = \beta_1 c_1 a_1 + \beta_2 c_2 a_2.$$

In this equality,  $a_i$  has been calculated at the optimum distribution of resources  $B_i$ . Put differently, in the above model the minimum average of migrants who have successfully overcome the border linearly depends on the corruption levels in oblasts at an optimum allocation of the

budget and a fixed distribution of the number of border guards. The above model shows how to allocate budgetary resources if the levels of corruption and the number of border guards are known for each oblast. For the two-oblast case, the solution was written out explicitly. Possibly, a model closer to reality would be one in which the number of border guards per oblast were not fixed, but ought to be allocated according to the budget. The local budgets of each oblast would in that case be subject to some lower constraint. However, a model of this kind could only be solved numerically.

## REFERENCES

1. *Borjas G. J.* (red.). Poverty, international migration and asylum / Borjas G. J., Crisp J. // Basingstoke, Palgrave Macmillan, 2005, 445 p.
2. EUBAM Annual Report 2005/2006. The European Union Border Assistance Mission to Moldova and Ukraine, Odessa, 38 p.
3. *Kyle D.* Global human smuggling: comparative perspectives. / Kyle D., Koslowski R. // Baltimore, Johns Hopkins University Press, 2001, 384 p.
4. *Пирожкова С.И.* Миграция и пограничный режим: Беларусь, Молдова, Россия и Украина./ Пирожкова С.И. и др. // Сб. Научных трудов, Центр изучения проблем вынужденной миграции в СНГ. Независимый исследовательский совет по миграции стран СНГ и Балтии, Киев, 2002, 176 с.
5. *Richards K.* The Trafficking of Migrant Workers: What are the Links Between Labour Trafficking and Corruption? / Richards, K. // International Migration, № 42(5), 2004, pp. 147-168
6. *Uehling, G.* Irregular and Illegal Migration through Ukraine. // International Migration № 42(3), 2004, pp. 77-109
7. *Vreja L. O.* Human trafficking in South Eastern Europe./ Vreja, L. O. // The Quarterly Journal, № IV(4), 2005, pp. 49-63
8. *Fischbacher U.* Strategic errors and audit quality: An experimental investigation. / Fischbacher, U, Stefani U. // The Accounting Review, № 82(3), 2007, pp. 679-704.
9. Serviciul grănicieri al Republicii Moldova, <http://www.border.gov.md>